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Summary Sheet

Saving Lake Mead: Recycling Wastewater Does Wonders

As a result of the increasing demand for natural resources, more and more areas are being transformed both directly and indirectly by man. Lake Mead, a reservoir that has the largest water capacity in the US, is calling for help. Therefore, we have analyzed the factors influencing the volume of water in Lake Mead, identified the patterns of the water level and the dry periods in Lake Mead, developed two different models for predicting the future water levels in Lake Mead, and prioritized the elements that should be included in a wastewater recycling scheme, measuring its impact as well.

We first found factors influencing the inflow, outflow, and loss and then analysed the relationships between factors and water levels by linear regression. The inflow is together determined by precipitation, temperature, and winds while the outflow is dictated by both residential and industrial consumption. As for the loss, it is mostly influenced by evaporation and temperature. The relationship between the factors and the water level is found by multivariable linear regression, which has been further confirmed with the available data.

Next, we predicted water levels for 2025, 2030 and 2050 using two different time series models based on our defined drought periods. We started with reviewing historical water level data trends for Lake Mead, identifying its overall pattern, and defined drought periods based on Meteorological Drought and Hydrological Drought derivatives with water levels as the primary indicator. The early and recent drought periods are compared in this section as well. Once the prerequisites were established, two models, Model A that employees Weighted Moving Average Method and Model B that applies Adaptive Filtering Method combined with Non-linear Regression, were well presented, with Model A using only recent drought water levels for prediction while Model B was based on all water levels from 2005-2020. The trends in water levels calculated by both models were generally consistent, with falling water levels dominating, but differences still existed in the specific data and periods. The predicted water levels by Model A, are generally lower than Model B, with an average annual water level of 734 feet in 2050. Model B, on the other hand, predicts the average annual water level to be 962 feet.

Ultimately, we used APH monolayer analysis to determine the priority of each element of a wastewater recycling plan to alleviate the water shortage in Lake Mead. We first concluded that Lake Mead may face serious water scarcity in the future and explored the usefulness of wastewater recycling and the factors it should include, such as the quality and volume of the wastewater and so on. With AHP, the priority of each factor will be determined by weighting, where the quality of the wastewater having the weighing of 0.41 came first. Finally, we presented a detailed plan for wastewater recycling and measured the impact of the plan from three perspectives: the environment, economics and society.

Keywords: Lake Mead, Water Level, Weighted Moving Average Method, Adaptive Filtering Method, Regression, Wastewater Recycling

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1. News Release

Saving Lake Mead: Recycling Wastewater Does Wonders

Since the summer of this year, the temperature in the western part of the United States has been higher than before, accompanied by widespread droughts. The most worrying thing is that Lake Mead Reservoir, the largest artificial lake in the western region of the United States, will also face a "red alert". In fact, the federal government has officially announced that the Lake Mead Reservoir is in a state of water shortage. According to relevant hydrological calculations, the storage capacity of the reservoir is only 36% of its full capacity, which is a record low since the completion of the Hoover Dam in 1930. If Lake Mead runs out of water, then Arizona, Nevada, New Mexico and other regions will significantly reduce water supply in 2022.

According to investigation and analysis from Team 11898, the water level of Lake Mead is mainly affected by factors such as precipitation, temperature, wind speed and wind direction. Concerning the temperature, the western part of the United States is experiencing extremely high temperatures recently, and there is not much rainfall or snowfall that can be left to prevent drought in the summer. As for the precipitation, which in the eastern region of Rocky Mountains mainly depends on the southwest monsoon, is also decreasing. This year, a huge ridge of high pressure appeared in the southwest of the United States, which is too far to the south, preventing the westerly wind and monsoon from reaching, causing less precipitation. This indicates that the recent climate will have a negative impact on the water level of Lake Mead.

Referencing to definition of Meteorological Drought and Hydrological Drought, with the water level being the indicator, the most recent drought period is found from April 2021 to present. The future water level of Lake Mead is predicted by Team 11898, finding that the water level of Lake Mead will continue to decline in the next 30 years.

In addition, the prediction shows that if the trend of the recent drought period continues, the water level by 2050 is only 740 feet, which is lower than the dead storage, 900 feet approximately, while if the trend of the water level from 2005 to 2020 lasts, the water level of Lake Mead in 2050 is predicted to be only 964 feet, which is also very close to the dead storage.

It is urgent to take measures to deal with the water shortage problem. In addition to reducing personal and industrial water consumption through government calls, a large-scale treatment of wastewater to effectively recycle wastewater resources is advocated. When implementing the wastewater recycling plan proposed by Team 11898, priority should be given to factors like the quality and volume of wastewater, construction and operation costs, and the difficulty of construction. The successful implementation of the wastewater recycling plan is expected to have a good impact on the local environment, economy and society.

Due to people's lack of awareness of water conservation in the previous century, coupled with climate factors such as today's greenhouse effect and global climate change, our freshwater resources have been severely reduced. As water resources are the source of life, we must cherish the existing freshwater lake resources and natural environment. We never know the worth of water till the well is dry. Therefore, we encourage everyone to save water, using water rationally and recycling water!

2. Introduction

Lake Mead, a Colorado River reservoir on the Nevada-Arizona border, is the largest water body and originates from the Hoover Dam on the Colorado River. In fact, 90 percent of the water resource in Las Vegas is derived from Lake Mead, which also provides the water system for around 40 million people across 7 states and northern Mexico[1].

However, in recent years, Lake Mead encounters an unprecedented drought, which straightforwardly causes a catastrophe to the whole water supply in the United States. In fact, the unrelenting drought that has had adverse effects on the Colorado River Basin for more than a decade is reducing the water resources which millions of people depend on, probably resulting in unavoidable water shortage issues in the west. From 2000 to 2020, Lake Mead has been significantly shrunk in size, represented by exposed rock formations that were originally below the water surface and thinner inflows.

Since Lake Mead is at risk of water shortage, it is necessary to analyse the existing patterns to predict the possible future situation of Lake Mead and, therefore, to provide possible solutions accordingly. By defining the drought period based on several criteria which take the water level as the most important indicator, the future of Lake Mead comes to life. Recycling wastewater, an approach which seems to be expensive and laborious, might be the key of alteration.

2.1 Question Restatement

First of all, we need to identify and describe the factors that affect the outflow, inflow, and loss of the lake and discuss the relationships between factors and their effects on the water level and storage. The next is to confirm the relationship between elevation, area, and volume in the official data table by describing the information and data needed as well as the mathematical process for calculating measures.

Secondly, the historical water level data for Lake Mead are supposed to be explored and it is also required to define the criteria for a dry period in Lake Mead. With the precondition of defining, the following step is to identify the beginning and end of drought periods and compare and evaluate the recent drought period with earlier ones. The final step is to develop and compare two prediction models for water levels as a function of the year, which predict the water levels of Lake Mead in 2025, 2030, and 2050, respectively.

Finally, based on the water level prediction models mentioned above, we ought to explain the impact of future water demand and consider whether wastewater reuse can compensate for the current water shortage. After identifying and describing considerations for planned wastewater use, priorities are listed and the possible impacts are evaluated.

2.2 Analysis of the Question

The essence of the problem can be divided into three major components: discussing the volume factors of Lake Mead, predicting Lake Mead’s water level trend in the future and considering the priorities concerning the wastewater recycling plan, and determining the ultimatum. These three questions can be broken down into the following sub-questions:

Question 1: We should first discover the factors which influence the inflow, outflow, and loss of Lake Mead. Moreover, it is indispensable to analyze the relationship among them and their connection to the water level and volume. The final step is to substantiate the estimation of the variables’ relationship.

Question 2: This task is to understand and describe the historical water level regularity of Lake Mead. Also, it requires defining the standard of drought duration and comparing early periods of drought to the most recent one. Furthermore, with data provided, the additional work is to take advantage of data in different time duration to establish two disparate prediction models of water level and contrast two models with one another with the ultimate evaluation.

Question 3: According to the prediction, the impact on future water demand and whether the recycling of wastewater can be an alternative to relieve the pressure of water scarcity should be further illustrated. Then, we need to take factors and priorities in the wastewater plan into account and evaluate the accessibility and impacts of the wastewater recycling plan.

3. Assumptions and Justifications

Assumption 1: We assume that the monthly water consumption of residents in the areas near Lake Mead is a constant within a year.

Justification 1: In a year, the monthly population change in areas around Lake Mead is relatively small compared to the total population, so the monthly population can be regarded as a constant. Meanwhile, each person's monthly water consumption is relatively fixed, so it is reasonable to assume that within a year the total monthly water consumption of all population in the areas is a constant.

Assumption 2: We assume that there will be no future extreme conditions that would cause dramatic changes in lake levels.

Justification 2: Extreme situations, such as the collapse of a dam caused by a strong earthquake bringing about drastic changes in the water level, have a very small probability of occurrence. Therefore, it is reasonable to ignore these extreme factors when establishing a model in this paper.

Assumption 3: We assume that recent climate patterns will continue in the future.

Justification 3: Although improvements in governments' policies and technological advancement may reduce future greenhouse gas emissions, we predict that it will take a long time for these projects to go from research to mature operation and significantly improve the ecological environment. At least from now to 2050, there will still be no major changes in climate patterns.

4. Symbol Notations

Symbol	Description
v_1	The real wind speed is decomposed into the wind speed in the southwest
E	Elevation of Lake Mead
ΔV	Changes in water volume of Lake Mead over a period of time
V	The total volume of Lake Mead
$\widehat{h_A}[t]$	Predicted water level of Lake Mead in the t^{th} month by model A
σ	Average relative error between predicted values and real values
$\widehat{h_B}[t]$	Predicted water level of Lake Mead in the t^{th} month by model B

$\widehat{W_{b_i}}$	Weight for the (t-i + 1)th month
w_{opt}	The best weight obtained

5. The Volume of Lake Mead

As the one of largest water reservoir, the volume of Lake Mead plays a critical role to the surrounding ecosystem and the population. Therefore, understanding the factors affecting the supply and consumption of Lake Mead's water resources allow us to better analyse the current state of it. Besides, the relationship between Lake Mead's volume of water and the height of the water table and the area of the region also pave the way to the future analysis.

5.1 Impacting Factors of the Inflow, Outflow and Loss

5.1.1 Relationships between Factors

As Lake Mead is an extremely important water supply location, it is crucial to first understand what factors affect its volume, which is subdivided in to inflow, outflow, and loss sections.

In terms of the inflow, precipitation, wind velocity, which has interrelation with evaporation, and wind direction, are determined to be the most influential elements while outflow is determined by both residential and industrial consumption. The loss, on the other hand, is mostly influenced by the evaporation and temperature with an interrelationship between each other. The illustration of the factors and their relationships are shown in **Figure 1**.

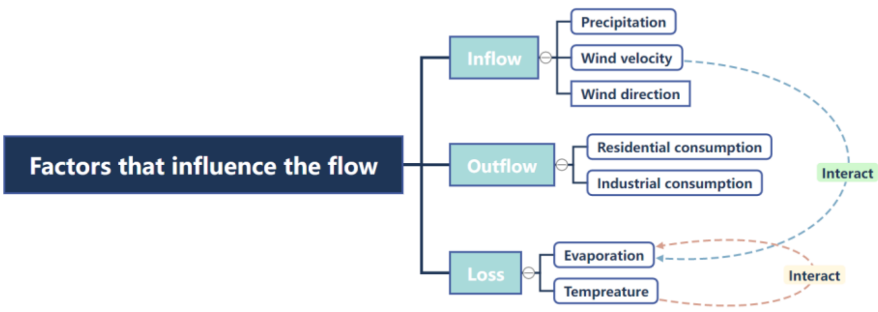


Figure 1. Illustration of Factors' Relationships

As for precipitation, many river basins in the United States, such as Lake Ontario and Lake Erie, are largely subject to the influence of precipitation. For example, precipitation causes Lake Ontario to suffer from very high and very low water levels, accounting for 15% of Lake Erie's water input[2]. For Lake Mead in the desert, the rare precipitation is a significant recharge, compensating for the usual evaporation of the amount of water.

Wind velocity and direction, on the other hand, determines the magnitude of inflows from other rivers. The wind has a huge impact on the Colorado River and other tributaries account for a large part, up to 96%, of the inflow. If wind speed and current velocity are in the right direction to blow water from the river into the lake, the input from other rivers will be significantly increased and vice versa.

Outflow and loss both lead to a reduction in water volume, except that outflow is biased towards anthropogenic water use and loss is influenced by natural factors. The anthropogenic water use can be broken down into industrial and residential water use, which are the two main reasons why human-being utilize water. Loss, considering that Lake Mead is located in the desert, is mainly affected by evaporation. Temperature plays a less significant role here, but it has an inseparable effect on water loss rates.

5.1.2 Factors' Influence on the Volume and Water Level

In order to better measure the impact of the above-mentioned factors on inflow, outflow, and loss, several equations and a Multivariable Linear Regression Model is applied.

To begin with, data on Lake Mead evaporation, precipitation, wind speed and direction, and temperature are required to be collected. Additionally, the residential water use and industrial discharge also need to be quantified.

However, the actual evaporation cannot be easily accessed, requiring other approach to calculate. The worth noting interrelationship between evaporation, wind speed and temperature appears to be the solution, which is explained by Dalton's formula for the law of evaporation of gases, and is shown as

$$W = C\frac{(E-e)}{P} \tag{1}$$

where W is the evaporation rate, C is the wind speed influence factor, the $(E-e)$ value is influenced by the humidity and temperature, and P is the atmospheric pressure, which can be considered as a constant.

Since evaporation is determined by wind speed and temperature, it makes sense not to include evaporation as an independent variable in the multiple linear regression model and the model can be established with temperature, precipitation and decomposed wind velocity being the three independent variables.

By establishing a multiple linear regression equation, the relative effect of each factor on the water level and volume of Lake Mead can be determined, and through accessing the official website of the US Weather Bureau, daily temperature and precipitation data as well as wind speed and direction data can be collected for 2019, partial data is show in **Figure 2.**

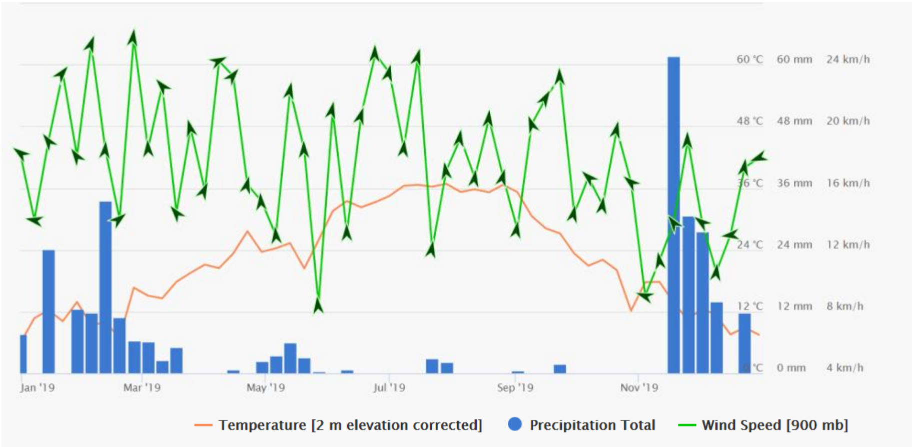


Figure 2. Daily Variations in Temperature, Precipitation and Wind speed and Direction in 2019[3]

As only 2019 data is accessible, 2019 Lake Mead water level data will be chosen for the Multivariable Linear Regression analysis with data of the mentioned factors.

Firstly the daily temperature, precipitation, and wind factors need to be pre-processed. Considering that the time unit of measurement for water levels is once a month, the daily temperature and precipitation data need to be converted into monthly averages.

Since the Colorado River, the main water supply for Lake Mead, is located to the northeast of Lake

Mead, winds blowing to the southwest have a boosting effect on the Lake Mead inflow, causing water levels to rise while winds blowing to the northeast have a dampening effect on Lake Mead levels, which is illustrated in **Figure 3.**

Figure 3. Illustration of the Decomposition of a Wind Vector[4]

Therefore, in order to clarify the specific effects of wind direction, winds are regarded as vectors in out models. We stipulate that the wind speed is positive in the direction to southwest and negative to the northeast after the decomposition of vectors. Therefore, the decomposition process can be described as

$$v_1 = \cos(\alpha - \frac{5}{4}\pi)v, \alpha \in [0, 2\pi] \tag{2}$$

where v_1 is the wind speed after the vector decomposition, the original wind speed is represented by v and its corresponding wind direction degree is α.

Due to the different order of magnitudes of temperature, precipitation and wind speed, each data needs to be normalized first in order to ensure the correctness of the regression process by

$$\widehat{x_i} = \frac{x_i - min\{x_1, \dots, x_{12}\}}{max\{x_1, \dots, x_{12}\} - min\{x_1, \dots, x_{12}\}} \tag{3}$$

where $\widehat{x_i}$ is the normalized value while x_i indicates corresponding data of the temperature, precipitation or wind speed in the i^{th} month.

Utilizing the data described above, the regression process can be carried out and the result is

$$E = -0.39T + 0.06P + 6.92v_1 + 1097.87 \tag{4}$$

where E is the monthly water elevation and T, P and v_1 represent the temperature, precipitation and decomposed wind velocity respectively.

The visualization of comparison between the fitted and actual values is shown in **Figure 4.**

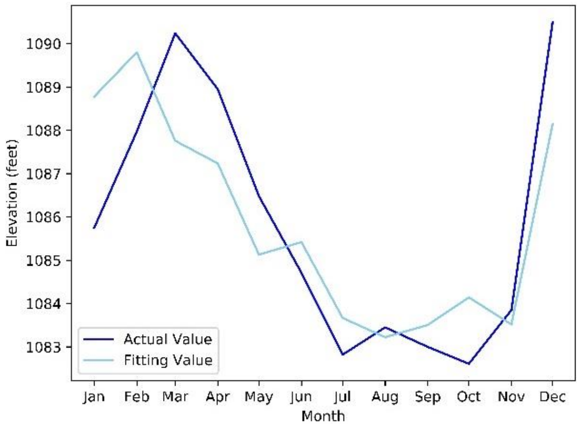


Figure 4. Comparison between Fitting and Actual Values of Monthly Water Elevation in 2019

It is obvious that the fitted data and the actual data follow approximately the same trend, indicating that the relationship between the water level and the factors is consistent with the pattern.

The factors' influence on volume utilizes the equations presented above. If it is assumed that the effect of these factors on the water level is small over a short period of time, the change in water level volume can be approximated as a column, which can be described as

$$\Delta V = S\Delta E \tag{5}$$

where the surface area of Lake Mead at elevation E is S, ΔE is the difference in water level over the period of time and ΔV is the corresponding volume difference.

In conclusion, the water volume and level of Lake Mead are both negatively correlated with temperature and positively correlated with precipitation and winds. Besides, the coefficients of the regression function show that winds have the greatest effect on water level, followed by temperature and then precipitation.

5.2 Data Relationship between the Elevation, Area and Volume

Although some of the data relationships for Lake Mead are listed, the relationships and plausibility of the data are not all well documented. Thus, we will analyse in detail the relationship between water level, area and volume of Lake Mead, verifying the potential data relationships.

5.2.1 Division of Lake Mead

Lake volume refers to the volume of a body of water or the volume of a lake basin below a certain water level. Due to the irregular shape of Lake Mead and the complexity of the geomorphology at the bottom, there is no way to calculate the volume of water directly. We therefore decided to divide the entire Lake Mead into sections to calculate the volume of water.

The highest point of the projection at the bottom of Lake Mead will be used as the horizontal division line between the upper and lower parts - the upper part can be consider as a irregularly shaped pedestal body with upper and lower bases and the lower part is seen as a collection of many triangular cones and quadrangular prisms, shown in **Figure 5**.

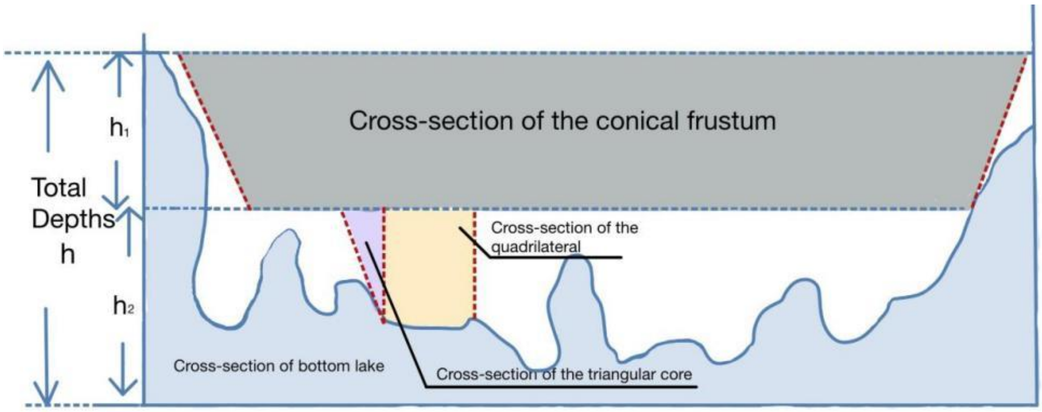


Figure 5. Sketch of the cross-section of Lake Mead

The reason for using a truncated cone as the superstructure of the model, rather than a cylinder with a constant area above and below, is that the slope of the shore causes the top radius of the water to be longer than the bottom radius underwater, resulting in a rectangular cross-sectional area. As the upper part of the lake is not as rugged as the lower part, similar to the formula for calculation the volume of a truncated cone, the volume of the upper part can be calculated by

$$V_{cf} = \frac{1}{3}h_1(S_1 + S_2 + \sqrt{S_1S_2}) \tag{6}$$

where h_1 is the height of the upper part, and S_1 and S_2 are the area of the upper and lower surface of the part respectively.

The reason for using trigonometric and quadratic prisms for the lower part of the lake is that the bottom of the lake forms different grooves depending on the height of the terrain. Some of these grooves have flat bottoms and some have sharp angles, which matches the base of the trigonometric and quadratic prism. Therefore, it is reasonable to use trigonometric cones and quadrangular prisms as tools for measuring the volume of river bottoms, and the total volume is

$$\begin{cases} V_{it} = \frac{1}{3} h_t S_b \\ V_{jq} = \frac{h_q(S_U + S_b)}{2} \end{cases} \tag{7}$$

where V_{it}, V_{jq} are the volume of the i, jth trigonometric or quadrilateral respectively; h_t, h_q are the heights of the trigonometric and quadratic prism respectively; S_U, S_b stand for the upper surface and bottom surface respectively.

By combining (6) and (7), the total lake volume can be obtained by

$$V = \sum_{i=1}^{n} V_i + \sum_{j=1}^{n} V_j + V_{cf} \tag{8}$$

where V is the total volume of Lake Mead.

5.2.2 Relationship Verification

By (8), the total volume of water in the Lake Mead reservoir can be determined and the next step is to verify the relationship between the data provided in **Table 1**, including the data of elevation, area of the lake, and volume of the lake.

To verify whether the model of the upper water section accords with the facts, data in the adjacent rows of elevation and area listed in the official data table are subtracted and then brought into (6) to calculate the volume of the upper part. If the difference between the calculated volume and the volume obtained by subtracting the adjacent data is relatively small, our model for describing the relationship between elevation, area and volume is proven to hold for the upper water volume. The outcomes are presented in **Table 1**.

Table 1. Comparison of Data Calculated by the Model and that Provided by the Bureau of Reclamation

Evaluation difference (feet)	1229-1219.6	1219.6-1050	1050-895
Official volume difference (acre-feet)	1456324	18012331	7641004
Estimated volume difference (acre-feet)	1563336	18620616	7688658
Difference between model data and official data	7.35%	3.38%	0.62%

It is evident that the differences between the model estimation and the official data are small - the smallest difference is even 0.62%. Therefore, it is reasonable to assume that the relationship described by (6) exists for the upper water table

The relationship in the lower part of Lake Mead, however, is hard to be verified since the data on the geomorphology of the bottom of Lake Mead, rough and tumble potholes, is lacking. In order to gain a more accurate result, the underground look which relies on utilizing sonar to detect is required. By rearranging the bottom area and fitting the appropriate trigonometric and quadratic prisms in, the lower volume of water body can be therefore calculated and verified.

6. The Water Level of Lake Mead

After the factors that influence the water level and volume of the lake are determined, we have had a general understanding of how the whole system works and interact. Therefore, concerning the drought situations, evaluating the current trend of water level and predicting the future trend of it is practical at this stage.

6.1 Analysis of Historical Water Levels of Lake Mead

In order to have a better understanding of what is going to happen in the future, it is not unreasonable to have a general overlook of the situation that Lake Mead has faced. Therefore, the overall water level pattern and our definition and findings of the drought periods which have occurred, are well demonstrated in the following sections.

6.1.1 Overall Water Level Patterns of Lake Mead

To see the trend in water levels, we first plotted historical monthly water elevation and yearly high and low water elevation in **Figure 6** and **Figure 7.**

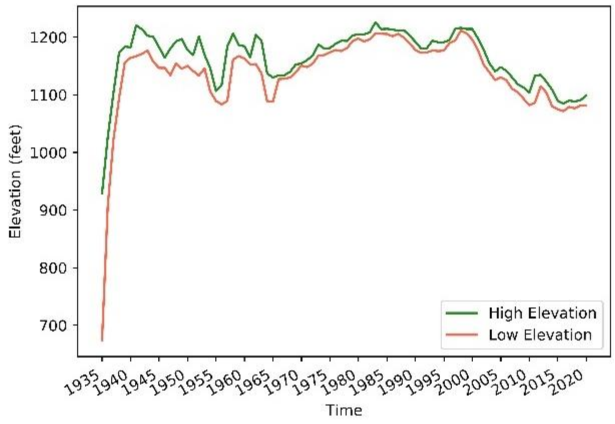


Figure 6. The Highest and Lowest Water Levels

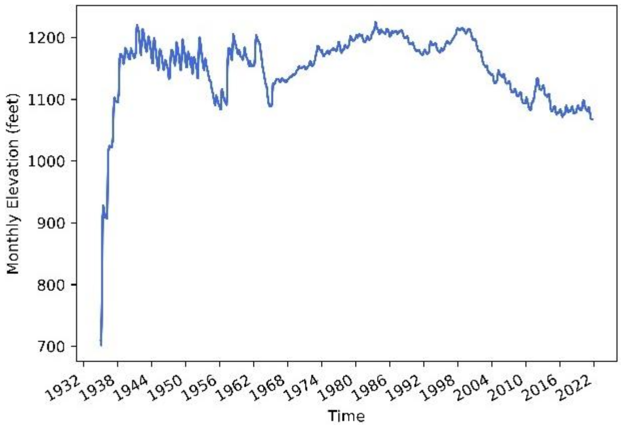


Figure 7. Monthly Average Water Levels

The general trends in the two graphics are substantially the same. The whole trend can be segmented into several time intervals according to the change in slope. In the initial state, a significant upward trend happened in the first five years, 1935-1940, which can be explained by the process of flooding - the abundant influx of water naturally increases the water level of the lake. The following stage is a bumpy downward trend during 1940-1955 and a pronounced upward trend occurred during 1955-1960, while the water level of 1960-1965 shows a downward trend with a greater difference between high and low water levels, but still maintaining the general agreement with the average water level.

Up to 1965-2000, there is a gentle upward trend in the water level, but from 2000-2020 the water level continues to fall and the highest level is a record low because of the growth in human water demand and weather changes.

Lake Mead's water level drops dramatically in some years for different reasons. During 1963, Glen Canyon Dam held back the water of the Colorado River that largely supplied Lake Mead to form Lake Powell, causing the elevation of Lake Mead to decrease drastically [5]. It was not until 1983 that Lake Mead's water level returned to full pool status. It is also worth noting that the steady decline that began at 1999 was due to continued dry weather and increased water consumption owing to population growth.

6.1.2 Criteria for Drought Periods

After a brief analysis of the overall pattern of the water levels of Lake Mead, we can then determine the drought periods of the lake for a more detailed investigation of the lake.

Two distinguished and widely accepted definitions of drought are Meteorological Drought and Hydrological Drought [6]. The former defines drought as a "prolonged period with less than average precipitation" while the latter claims that drought takes place when water reserves, reservoirs for instance, fall below a "locally significant threshold". Therefore, our definition of drought periods combines the mentioned two criteria and uses the water level as the indicator.

Employing the similar measure used in Meteorological Drought definition, which compares current precipitation with previous average precipitation, our first drought criterion compares the water level of a selected month with the average water level of the previous three months. If the water level in the selected month is lower than the average of the previous three months, the month is considered to be part of a drought period.

The idea of Hydrological Drought is summarized in our criteria by finding a water level that declares Lake Mead to be water scarce. In 2015, the news indicated that the Bureau of Reclamation announced the drought warning when the water level of Lake Mead went below 1085 feet [7]. Thus, it is reasonable to set 1085 feet as the threshold.

A period of drought can be declared as long as it satisfies both the criterion of being less than the threshold, 1085 feet, and the criterion of being below the average water level of the previous three months. Therefore, the starting month, ending month, and duration of a drought can be found, and are presented in **Table 2,** in which the periods highlighted in orange are the drought periods.

Table 2. Starting Month, Ending Month, and Duration of Drought Periods

Year	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1956	1089.2	1087.1	1083.6	1083.8	1098	1116.6	1114.6	1110.7	1105.8	1102.1	1099.1	1097
2014	1108.8	1107.9	1101.7	1094.6	1087.5	1082.7	1080.6	1081.6	1081.3	1082.8	1083.6	1087.8
2015	1088.5	1089	1084.9	1079	1076.6	1075.1	1078.2	1078.3	1078.1	1079	1078.2	1080.9
2016	1083.7	1084.2	1080.5	1076.1	1073.8	1071.6	1072.8	1075.2	1075.2	1076.3	1076.6	1080.8
2017	1086.1	1089.8	1088.3	1084.9	1081.6	1079.5	1079	1081.4	1082.1	1082.3	1081	1082.5
2018	1087.5	1088.2	1088.1	1084.5	1080	1076.8	1077.4	1078.9	1078.3	1078.5	1078.3	1081.5
2019	1085.8	1088	1090.2	1089	1086.5	1084.7	1082.8	1083.5	1083	1082.6	1083.9	1090.5
2020	1094.7	1096.3	1098.6	1096.4	1091.3	1087.1	1084.6	1084	1083.2	1081.9	1081.1	1083.7
2021	1086	1087.3	1084.4	1079.3	1073.5	1068.8	1067.7	1068	1067.7			

6.1.3 Comparison between Drought Periods

The frequency of drought periods has increased since 2014, with shorter intervals between drought occurrences compared to previous drought periods.

Recent drought periods have a longer time duration, with the previous drought period, which happened in 1956, lasting for only two months. Recent drought periods typically last for six consecutive months before a shift occurs. The trend was most pronounced in 2015 onwards. The situation is getting worse year by year: in 2015, there were three months of drought and a drought period of four consecutive months happened in 2016 while year 2017 was with a drought for five consecutive months.

Trends of water levels during early drought periods and recent drought periods are plotted in **Figure 8.**

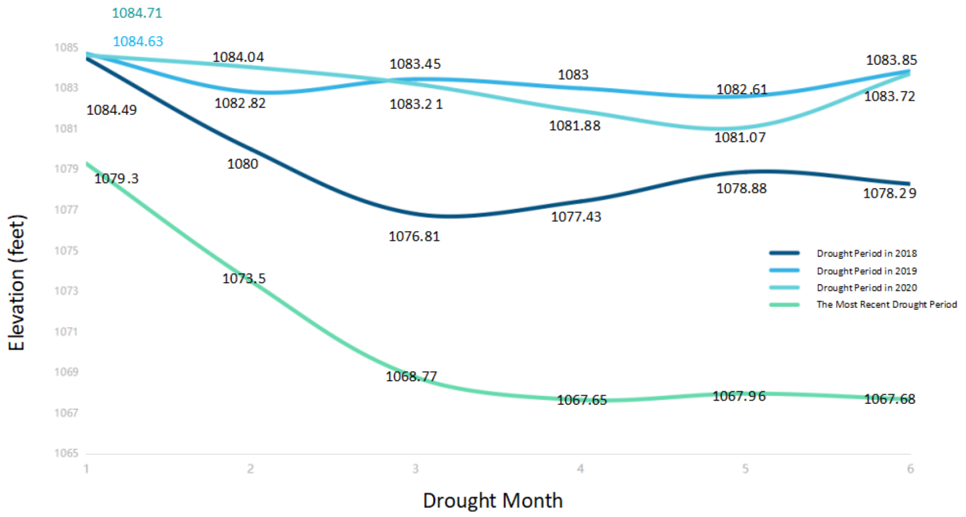


Figure 8. Comparison of Water Levels between the Earlier and Recent Drought Periods

The figure shows a significant decreasing trend in water levels during the recent drought periods, and the elevation values of the most recent drought period are extremely lower than those of the previous drought periods, indicating that the drought has been more and more severe as time goes by.

6.2 Prediction of Future Water Levels of Lake Mead

After having had a clear overview and analysis of the historical water level data of Lake Mead, the next step can be taken: predicting future water level trends. By using the set of data described in the previous sections, two prediction models, Weighted Moving Average Method and Adaptive Filtering Method combined with Non-linear Regression, were therefore proposed. Further, the two models are also compared and evaluated in this section.

6.2.1 Model A: Weighted Moving Average Method

The basic idea of the Weighted Moving Average Method, which is one of the time series models, is to give greater weights to recent data considering the different importance of the data at different times. This appropriately fits in the situation of a more similar climate pattern between consecutive months than months with long intervals and the context of utilizing few data to make predictions.

The data of the most recent drought period, from April to September in 2021 indicated in **5.1.2**, is processed as raw data for model A. Since the climate patterns of consecutive quarters, which includes three months, are relatively more similar, we consider that the climate of the predicted month is more likely to be related with three previous months. For the predicted month, the water level of last month is with a more internal relevance and greater weight, while the further apart the month the greater the difference in climatic conditions and the less closely the data are correlated. Thus, considering data from the previous three months before the month which is going to be predicted, the predicted data is

$$\widehat{h_A}[t+1] = \frac{w_{a_1}h[t] + w_{a_2}h[t-1] + w_{a_3}h[t-2]}{w_{a1} + w_{a2} + w_{a3}}, t = \{t \in Z, t \geq 3\} \tag{9}$$

where $\widehat{h_A}[t+1]$ is the predicted water level of the (t+1)th month; $h[t]$, $h[t-1]$ and $h[t-2]$ are the water levels in month t, month t-1 and month t-2; w_{a_1}, w_{a_2} and w_{a_3} are the corresponding weights, where $w_{a1} = 3$, $w_{a_2} = 2$, and $w_{a_3} = 1$.

For verification, we predicted the water levels for July, August and September in 2021 by (9). The predicted water levels are all generally larger than the actual water levels, which are shown in **Table 3**, proving that we need to adjust the predicted values by (9)**.**

Table 3. Actual and Predicted Values for July, August and September in 2021

Month	2021/7	2021/8	2021/9
The actual water level	1067.7	1068	1067.7
Model predicted water level	1072.1	1069	1068

The relative error σ between the predicted values and actual values is

$$\sigma = \left(\frac{\sum_{t=6}^{8} \widehat{h_A}[t+1]}{\sum_{t=6}^{8} h[t+1]} - 1\right) \times 100\% = \left(\frac{3209.1}{3203.4} - 1\right) \times 100\% \approx 0.18\% \quad (10)$$

Since the predicted values are averagely 0.18% higher than actual values, the predicted value $\widehat{h_A}[t+1]$ can by divided by 1+0.18%, with $\frac{\widehat{h_A}[t+1]}{1+0.18\%}$ serving as the corrected predicted values of model A.

Based on the weights obtained above and the proposed correction criteria, water level predictions for 2025, 2030 and 2050 are presented in **Table 4.a and 4.b.**

Table 4.a. Predicted Water Levels of 2025, 2030 and 2050 from January to June by Model A (feet)

Water level	January	February	March	April	May	June
2025 (year)	1023. 11	1022. 01	1020. 90	1019. 80	1018. 69	1017. 59
2030 (year)	958. 80	957. 76	956. 73	955. 69	954. 66	953. 63
2050 (year)	739. 51	738. 71	737. 914	737. 11	736. 31	735. 52

Table 4.b. Predicted Water Levels of 2025, 2030 and 2050 from July to December by Model A (feet)

Water level	July	August	September	October	November	December
2025 (year)	1016. 49	1015. 39	1014. 29	1013. 20	1012. 10	1011. 01
2030 (year)	952. 59	951. 56	950. 54	949. 51	948. 48	947. 46
2050 (year)	734. 72	733. 93	733. 14	732. 34	731. 55	730. 76

It can be clearly seen that from 2025 onwards, the water level has been below the threshold in our drought period criteria and continues to fall. It is even worse that the water level in 2050 is already well below Lake Mead's dead storage, 900 feet approximately, for the whole year. In other words, Lake Mead is likely to disappear shortly after 2050, which will bring enormous negative impacts for both humans and the ecosystem.

6.2.2 Model B: Adaptive Filtering Method with Non-linear Regression

The major difference between Model A and Model B is that Model B can use a larger amount of data, from 2005-2020, compared to Model A which can only use data from the most recent drought period. Therefore, the adaptive filtering method, which requires a larger amount of data to self-adjust the weights, ideally suited to the case. Adjustments of the weights allow the predicted value to be close to the true value in order to obtain the best weighs and the minimum error.

The process of adaptive filtering can be broken down into several specific steps. Firstly, a set of known data is given certain weights to calculate a predicted value and the error between the predicted values and the actual values is calculated. The corresponding weights are adjusted according to the error. This is repeated to obtain the 'best' set of weights to minimize the error. The predicted values are

$$\hat{y}[t+1] = \sum_{t=1}^{N} w_{b_i} y[t-i+1] \tag{11}$$

where $\hat{y}[t+1]$ is the forecast value for month $t+1$; $y[t-i+1]$ is the observation for month $t-i+1$; w_{b_1} is the corresponding weight and N is the number of weights.

The formula for adjusting the weights is

$$\widehat{W_{b_i}} = W_{b_i} + 2ke_{i+1}\, y[t-i+1] \tag{12}$$

where $\widehat{W_{b_i}}$ is the weights after calibration, $i = 1,2,...,N$; k is the learning rate and e_{i+1} is the prediction error at month $t+1$.

The equation for adjusting the weights shows that the adjusted weights should be equal to the old weights plus the error adjustment, which in turn includes three factors: the prediction error, the original observation and the learning rate which determines the speed of adjustment of the weights.

For the model mentioned above, the number of weights N needs to be determined first - in other words, how many months of data are relevant before the month being predicted. Since Lake Mead water levels are strongly influenced by climate, the climate for the predicted month will therefore be mainly similar to and more influenced by the previous three months similar to model A, assigning a value of 3 to N.

Once N has been determined, the water level prediction session can be performed. Then we can initialize k and the weights, where $w_{b_i} = \frac{1}{N}$. Next, the water levels for each month from 2005 to 2020 will be predicted by (11) and the error between the predicted and true values will be calculated and recorded. The following step is to determine whether the error has decreased by less than 1% compared to the previous error, if not, then the weights $\widehat{W_{b_1}}$ are updated and adjusted by (12).

It is worth noting that there is no comparison data for the first error and the weights will be updated directly by (12). This process, demonstrated in **Figure 9,** is repeated until convergence, and the current best weights can be obtained.

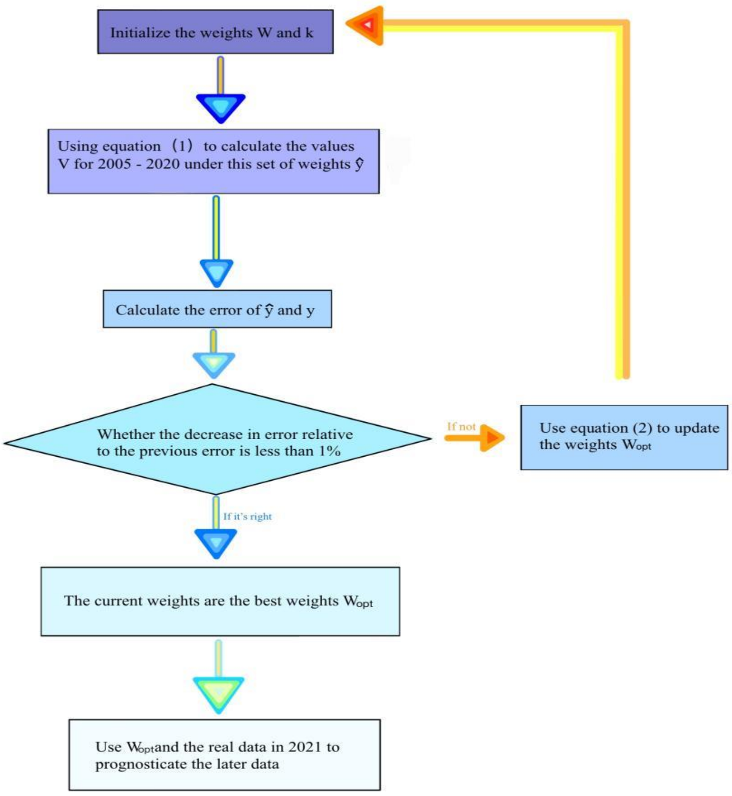


Figure 9. Flow Chart of the Adaptive Filtering Method

After several iterations of the process, the final calculated optimal weights are $w_{opt_1} = 0.3293$, $w_{opt_2} = 0.3327$, and $w_{opt_2} = 0.3374$.

When time series methods are used for long-term forecasting, the relationship between monthly water levels and months can become a simply linear relationship at later stages. However, it was observed that the water level varies cyclically within a year. Therefore, water level data from the most recent year, 2020, are used and fitted by a trigonometric function to the data within a year to compensate for the simple linear relationship that occurs when only using the time series method.

The 2020 data is fitted using the sine function which is

$$y = A\sin(\omega t + \theta) + B \tag{13}$$

The values of the parameters obtained after the fitting are shown in **Table 5** and the visual comparison of the fitted and actual values is shown in **Figure 10.** It is clear that the fitted data and the actual data follow approximately the same trend, indicating that our non-linear fitting of the values within a year is reasonable.

Table 5. Trigonometric function fitting parameter table

A	ω	θ	B
7.8663	0.4185	0.6089	1088.9191

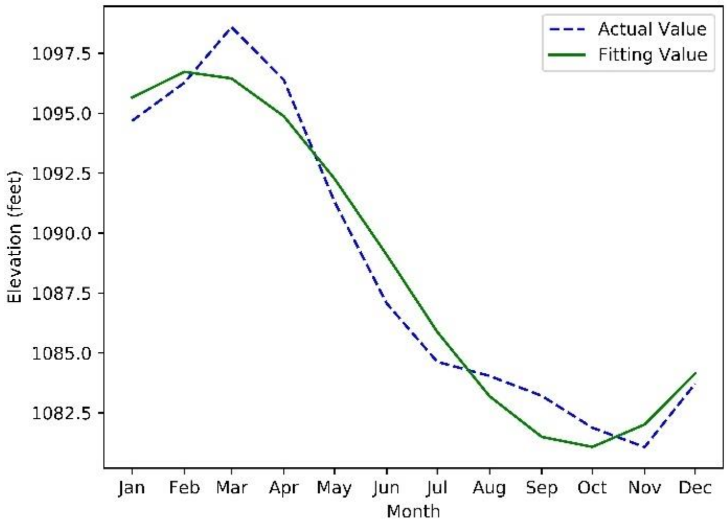


Figure 10. Comparison between Fitting Values by Non-linear Regression and Actual Values

Combining this fitted function with the time series method, the final prediction value of model B is

$$\widehat{h_B}[t] = \frac{y[t_1]}{y[t_0]}[A\sin(\omega t + \theta) + B] \tag{14}$$

where the final prediction of the water level of month t is $\widehat{h_B}[t]$; the water level in month t of a given year predicted by the time series method $y[t_1]$ and the water level of the same month t in 2020 is $y[t_0]$.

By combining the Adaptive Filtering Method with Non-linear Regression, we can obtain water level predictions for 2025, 2035 and 2050 that are consistent with both intra-annual cyclical variability and inter-annual trends, which are shown in **Table 6.a** and **Table 6.b**.

Table 6.a. Predicted Water Levels of 2025, 2030 and 2050 from January to June by Model B (feet)

Water level	January	February	March	April	May	June
2025 (year)	1056.53	1055.71	1052.9	1053.18	1055.23	1055.95
2030 (year)	1037.41	1036.61	1033.84	1034.12	1036.14	1036.84
2050 (year)	964.33	963.58	961.01	961.27	963.14	963.8

Table 6.b. Predicted Water Levels of 2025, 2030 and 2050 from July to December by Model B (feet)

Water level	July	August	September	October	November	December
2025 (year)	1054.89	1052.53	1051.36	1051.93	1053.31	1052.48
2030 (year)	1035.8	1033.48	1032.34	1032.89	1034.25	1033.43
2050 (year)	962.83	960.67	959.61	960.13	961.39	960.63

6.2.3 Model Comparison

Although Model A and Model B are homologous, empowered predictions based on water levels in the previous three months and both fall under the umbrella of time series models, there are significant differences in terms of methodology and results.

For Model A, the advantage is that future water level predictions can be made using less raw data. It also provides faster access to water level data for the forecast months and gives an approximate trend of water level changes. But the drawback of only using data from the most recent drought period is also apparent: the lack of a complete year data prevented a periodic fit to the water levels within a year. In fact, when we use the data from 2005 to 2020 to carry on the error test, Model A has an error rate of 0.4% while Model B has only 0.3%, verifying that Model B is indeed more accurate.

Model B, on the other hand, is more likely to give more accurate results because it uses a number of iterations of updating the weights by utilizing a large amount of data and constantly adjusting itself to get the 'best' weights to predict water levels. In addition, using time series for long-term forecasting leads to the result that the data can easily become a simple linear relationship at a later stage. Nevertheless, the simple linear relationship is compensated in Model B by fitting a trigonometric function to simulate the periodic variation, which greatly increases the predictability of the model. But because Model B requires a lot of data, it has a slower speed of calculation comparing to Model A, which is the flaw of Model B.

An overall comparison of the annual mean water levels predicted by Model A and Model B for the years 2025, 2030 and 2050 is shown in **Figure 11**.

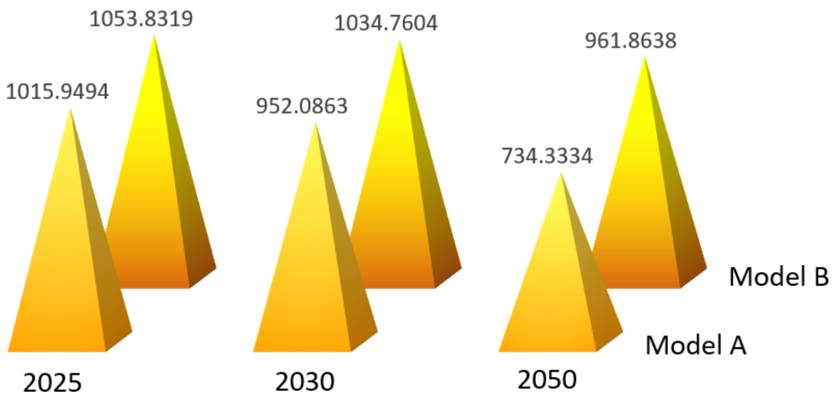


Figure 11. Overall Comparison of Two Models on the Annual Average Water Levels (feet)

It can be seen that Model B predicts a higher mean annual water level than Model A, and both models predict a decreasing trend in mean annual water levels.

The monthly predictions from two models for 2025, 2030 and 2050 are shown in **Figure 12**, **Figure 13** and **Figure 14** respectively.

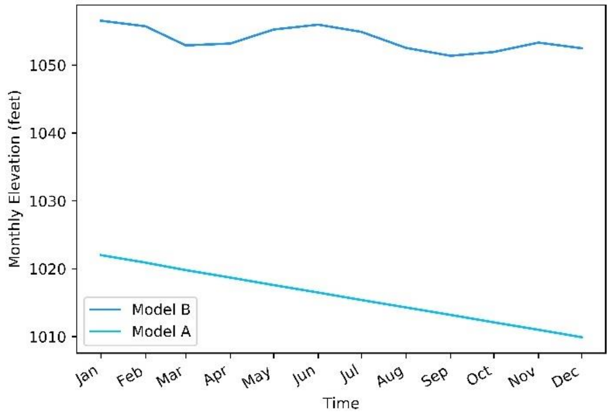
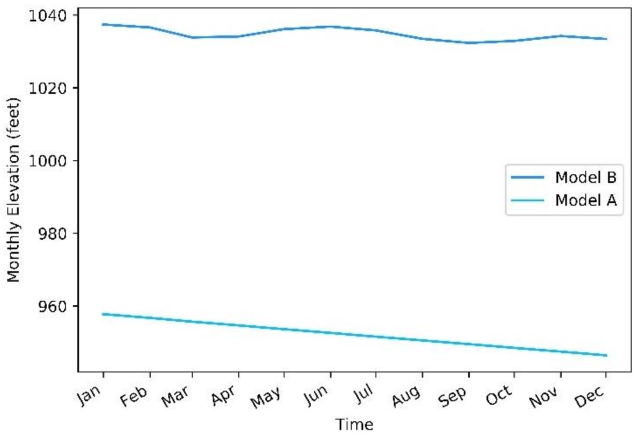


Figure 12. Monthly Water Level Predictions of 2025 from Two Models

It can be seen that not only the annual average water levels, but also the monthly water levels predicted by model B are greater than those predicted by model A. The water levels predicted by model B also display a certain cycle, while the water levels predicted by model A decrease in a linear manner.

Although Model B uses a non-linear fit to periodically adjust for the trend in water levels within a year, the predicted water levels remain progressively flatter as the year goes by. At the same time, while water level predictions from Model A consistently decreases linearly with month, the absolute value of the slope becomes smaller and smaller - caused by the variance of the time series failing to show a long-term trend and eventually converging to a linear relationship or a constant.



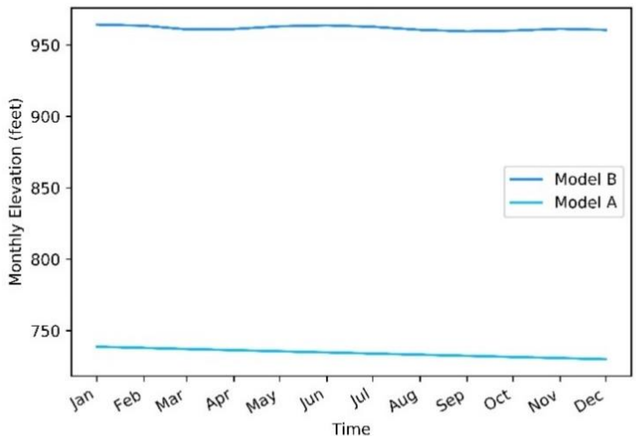


Figure 13 and 14. Monthly Water Level Predictions of 2030 (left) and 2050 (right) from Two Models

7. Wastewater Recycling Plan

Whichever prediction model is used, there is no doubt that the water level of Lake Mead has a tendency to keep falling. This means that tens of thousands of residents around Lake Mead will lose their vital water resources and struggle to survive in the near future. At the same time, the dams used for hydroelectricity generation will be abandoned, leading to a drastic reduction in available electricity. For the environment, prolonged drought will cause incalculable damage to the surrounding flora and fauna and lead to changes in soil quality.

This is where the wastewater recycling scheme is a lifesaver for Lake Mead in a drought situation. By offsetting the remaining water loss due to wind speed and temperature and gradually filling Lake Mead with water, waste water recycling will not only prevent the water level from dropping, but will also restore it.

7.1 Factors in Wastewater Recycling Schemes and Their Priorities

In order to improve the achievability of wastewater recycling schemes, a number of factors need to be taken into account in the scheme. Wastewater quality, plant construction and operating costs, ease of construction, local views on recycling and reusing wastewater, and volume of wastewater are the six key factors that need to be taken into account in this paper.

The quality of the wastewater and the volume of it can be consider as in the equally important status, having distinguished focus. In terms of water volume, deciding how much wastewater to treat in order to gradually raise the water level of Lake Mead is necessary. As for the water quality, determining the amount of wastewater treatment required sets the stage for all subsequent construction and planning, while testing the quality of the wastewater determines the means, acidification, oxidation biological, or sterilizing methods, etc. Therefore, it is determining the quality and volume of the waste water that facilitates the smooth process of the following procedures.

As for the cost and ease of construction, these two factors will be considered as the same priority. The location of the wastewater treatment plant, the materials required for both the construction and the human resources are all factors to be considered.

The public's perception of wastewater recycling is also important. If residents refuse to provide wastewater or do not agree with the wastewater treatment plan, then it is difficult to implement the whole scheme. The priority of public opinion is relatively lower than for the previous elements because by constantly emphasizing the dangerous situation facing Lake Mead and the importance of wastewater recycling, there is a high probability that the public will understand and accept the wastewater recycling scheme.

To more scientifically prioritize the various factors, Analytic Hierarchy Process (AHP), developed by Santy[8], will be applied. Employing Santy's 1-9 scale method, which is shown in **Table 7.** to form a pairwise comparison matrices, enables us to quantify the priority of each factor in the wastewater recycling scheme.

Table 7. Santy's 1-9 Scale Method

	Rating	Importance
Scale	1	The two factors are equally important
	3	One factor is more important than the other
	5	One factor is strongly more important than the other
	7	One factor is very strongly more important than the other
	9	One factor is extremely more important than the other

The pairwise comparison matrix represents a comparison of the relative importance of all factors in this layer against a factor in the previous layer, and the comparison matrix of our model is shown in **Table 8.**

Table 8. Comparison Matrix of Factors in the Wastewater Recycling Plan

	Quality of the wastewater	Cost of the construction	Volume of the wastewater	Ease of the construction	Public's perception
Quality of the wastewater	1	3	2	4	5
Cost of the construction	$\frac{1}{3}$	1	$\frac{1}{2}$	2	3

Volume of the wastewater	$\frac{1}{2}$	2	1	3	4
Ease of the construction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	1	2
Public's perception	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	1

By finding the maximum eigenvalue and the corresponding eigenvector in the matrix, determining whether it passes the consistency test, and normalizing the eigenvector, the final weight vector is obtained and shown in **Table 9**.

Table 9. Weights of Factors in the Wastewater Recycling Plan

Destination Layer	Criterion Layer	Weight
The most influential factor in the wastewater recycling plan	Quality of the wastewater	0.4185
	Cost of the construction	0.1599
	Volume of the wastewater	0.2625
	Ease of the construction	0.0973
	Public's perception	0.0618

Depending on the weights of each factor, the wastewater recycling scheme can be prioritized, with quality of the wastewater came first and public's perception came last, which is visualized in **Figure 15.**

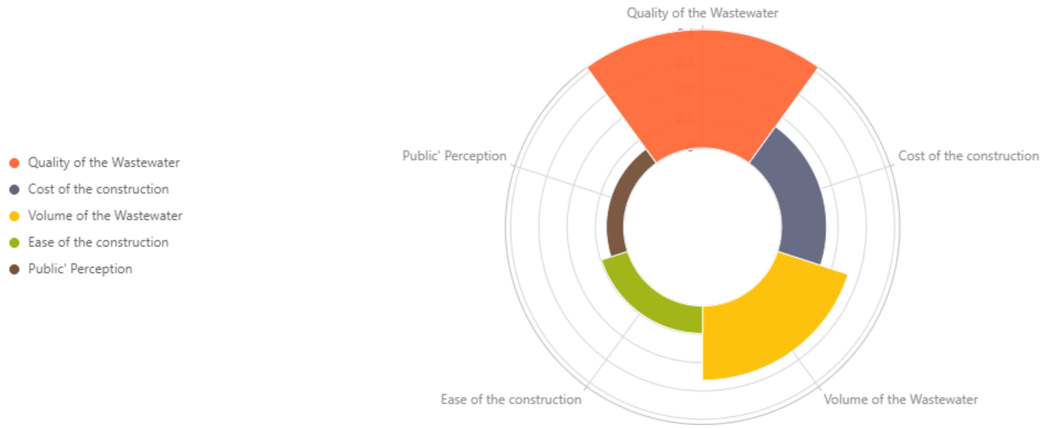


Figure 15. Rose diagram of weight for criteria

7.2 The Waste Water Recycling Plan and Its Future Impacts

In our plan, we believe that wastewater recycling needs to undergo two main treatments: physical treatment and chemical treatment, which are shown in **Figure 16.**

For the physical treatment, the wastewater is first introduced to the bottom of the tank and passed through a sludge filter for the treatment of impurities. The first filtered effluent will then be diverted: at the bottom of the scum filter zone the effluent will be discharged underground, while in the middle an outflow pipe will be connected to allow the filtered, scum-treated water to flow out for agricultural irrigation purposes.

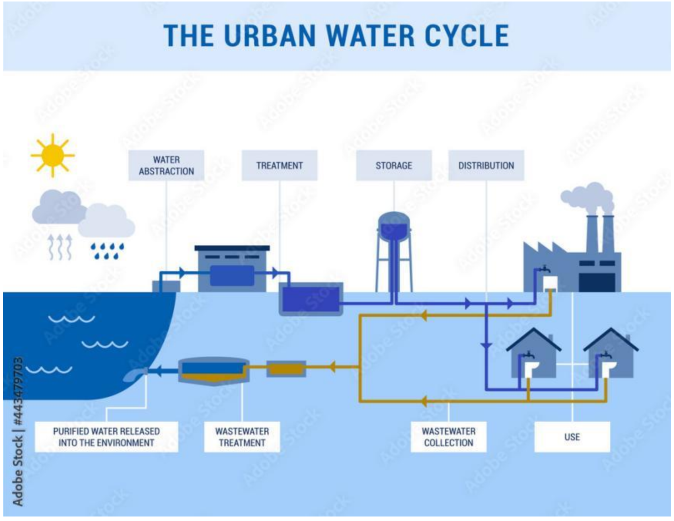


Figure 16. Illustration of Wastewater Process[9]

On the other side of the sludge filtration zone there will be connecting pipes. These pipes have an infiltration system built into them, which allows the first filtered waste water to flow into the drainage ditch of the drainage system for secondary filtration of the effluent.

We also have small layers of pebbles and quartz gravel at the bottom of the drains. These stone layers filter the effluent entering the drains for a third time and allow for secondary sedimentation.

Finally, the treated water is introduced into the reservoir for chemical treatment. Alum is added to the water in the right dose and stirred to adsorb the tiny particles that were not previously filtered out. Activated carbon is then added to adsorb pigments, odors and finally chlorine is added to give the water a final disinfection and sterilization treatment to ensure it is clean.

In terms of measuring the impact of implementing, our wastewater recycling plan mainly considers three aspects, which are related to the environment, economics and society respectively.

The environmental impact is the most direct impact that can be noticed after the implementation of our plan. We will use the concentration of the main pollutants in the treated wastewater to measure the environmental impacts brought about by the plan. After the wastewater treatment, the concentration of the main pollutants like SS, NH_3-N, etc. in the wastewater will be reduced to varying degrees, greatly reducing water pollution caused by urban sewage discharge. The plan will play a positive role in the restoration and improvement of the ecological environment in the region with reduced pollutants' concentration in wastewater.

Taking into consideration the impact on the economy, in the initial stage, the wastewater recycling plan may require lots of funds and it is difficult to see direct economic benefits. However, after the successful implementation of the plan, the recycling of wastewater can reduce the economic losses caused by sewage in the region, which can be measured in the following three aspects[10].

First, the wastewater recycling plan will avoid deterioration of water quality, thereby reducing economic losses caused by damage to urban water supply facilities caused by water pollution. Secondly, the plan can reduce the decline in the quantity and quality of agricultural, animal husbandry, and fishery products caused by water pollution and bring certain economic benefits. Finally, the plan can reduce the possibility of water pollution causing a decline in the health of urban residents, thereby reducing residents' corresponding health care costs.

The improvement of the overall image of the city and the increase of jobs can reflect the impacts on the society. For one thing, after the implementation of the wastewater recycling plan, the city's sewage treatment facilities, infrastructure and other construction will be improved, thereby improving the city's image and increasing the city's overall competitiveness. For another, the plan will provide various employment opportunities such as technical workers to alleviate the employment tension in the society.

8. Strengths and Weaknesses

8.1 Strengths

- The factors affecting inflow, outflow and loss in Lake Mead have been discussed in detail in this paper. We have carried out the corresponding vector decomposition of directional factors such as winds, so that more accurate results can be obtained when quantifying their influence on the volume and water level of Lake Mead.
- Based on the different amounts of data available, we chose different time series forecasting models to predict the future water level, and considering the limitations of the time series forecasting model, we innovatively combined the nonlinear fitting and time series forecasting to predict the future water level. Therefore, the predicted results can not only describe the trend of the water level change from year to year, but also the cyclical fluctuations of the water level within a year.
- When discussing the influence of various factors, we have achieved a simultaneous analysis from both qualitative and quantitative aspects. For the factors that affect Lake Mead volume and water levels, and wastewater recycling plan, we first described the relationships between the factors qualitatively, and then used linear regression and AHP to quantitatively describe them respectively.

8.2 Weaknesses

- Although our definition of drought combines two widely accepted drought standards with certain theoretical basis, we mainly use monthly water elevation values to judge whether there is a drought period. We have not taken into account factors such as precipitation and residents' water consumption, which may not be comprehensive and exhaustive enough.
- Our forecasting models are mainly based on time series forecasting. Although we used the nonlinear fitting and other corrections, in the long run, the time series forecasting model may not be accurate. More prediction models such as neural networks may be able to provide more accurate prediction results.
- Our predictions for future water levels are all monthly elevation predictions. There are no corresponding high and low water level predictions, which may not be comprehensive enough to describe the drought trend in the next few years in detail.

9. Model Sensitivity Analysis

Model sensitivity analysis is necessary in order to measure whether the developed models is adaptive to changes in a number of factors.

In Models A and B, the predicted water levels were assumed to be primarily related to the water levels of the first three adjacent months, which is $N = 3$. To test the sensitivity of the model, we now change N to 5 and then make predictions for the 2025 water level data, which is shown in **Figure 17**.

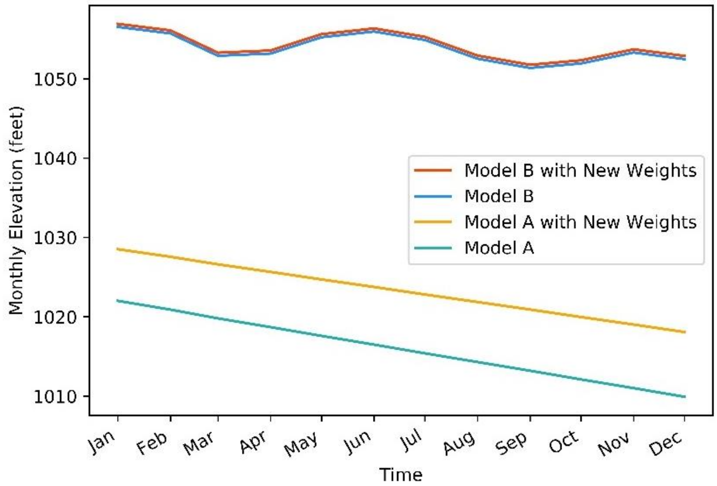


Figure 17. Sensitivity Analysis and Comparison of Model A and Model B

The graph shows that although there is some increase in the new water levels, the overall difference is not significant in Model B, however, the difference after the change is relatively larger in Model A. It depicts that Model B has a better ability to adapt to changes in factors since it is more closely aligned with the original predicted water levels.

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Appendix

1. Python Code for Linear Regression

```
import numpy as np
import pandas as pd
import statsmodels.api as sm
from datetime import datetime
import matplotlib.pyplot as plt
import matplotlib.dates as mdates
# Data Preprocessing
water_level = [1085.75, 1087.97, 1090.24, 1088.95,
1086.48, 1084.71, 1082.82, 1083.45, 1083, 1082.61,
1083.85, 1090.49]
temperature = [10.2440, 10.4012, 16.8821,
23.0472, 23.6659, 32.3907, 36.0474, 35.8043,
30.218, 20.1081, 15.0855, 9.8487]
precepitation = [32, 68.5, 20.1, 3.5, 12.7, 0.8, 5.1, 0,
2.3, 0, 92.3, 53.2]
wind_avg = [-0.44618, 0.04466, -0.33012, -0.21177,
-0.39705, 0.02506, 0.0166, -0.10798, -0.35953, -
0.85545, -0.3821, -0.36636]
data_x = np.array([temperature, precepitation,
wind_avg]).T
max_x = np.max(data_x, axis=0)
min_x = np.min(data_x, axis=0)
data_x_norm = (data_x - min_x) / (max_x - min_x)
data_y = np.array(
   [[1085.75, 1087.97, 1090.24, 1088.95, 1086.48,
1084.71, 1082.82, 1083.45, 1083, 1082.61, 1083.85,
1090.49]]).T
# Linear regression
mod = sm.OLS(data_y, sm.add_constant(data_x))
res = mod.fit()
print(res.summary())
data_y_fitting = -0.3883 * data_x.T[0] - 0.0634 *
data_x.T[1] + 6.9235 * data_x.T[2] + 1097.8734
# Plotting
date_list = [x.strftime('%Y-%b') for x in
list(pd.date_range(start="2025-Jan", end="2026-
Jan", freq='M'))]
x_date = [datetime.strptime(d, '%Y-%b') for d in
date_list]
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
ax.xaxis.set_major_formatter(mdates.DateFormatt
er('%b'))
locator = mdates.MonthLocator()
ax.xaxis.set_major_locator(locator)
plt.plot(x_date, data_y, color='blue', label='Actual
Value')
plt.plot(x_date, data_y_fitting, color='skyblue',
label='Fitting Value')
plt.legend()
plt.xlabel('Month')
plt.ylabel('Elevation (feet)')
plt.show()
```

2. Python Code for Weighted Moving Average

```
import numpy as np
his_data = [1079.3, 1073.5, 1068.8, 1067.7, 1068,
1067.7]
total_months = (2050 - 2021) * 12 + 6
predict_data = his_data.copy()
year_2025 = []
year_2030 = []
year_2050 = []
for i in range(total_months):
  if i < 3:
     pred_elev = (his_data[i] + 2 * his_data[i + 1] +
3 * his_data[i + 2]) / 6
  elif i == 3:
     err = (predict_data[3] + predict_data[4] +
predict_data[5]) / (his_data[3] + his_data[4] +
his_data[5]) - 1
     # print("err:", err)
     pred_elev = (his_data[i] + 2 * his_data[i + 1] +
3 * his_data[i + 2]) / 6
  elif i == 4:
     pred_elev = (his_data[i] + 2 * his_data[i + 1] +
3 * predict_data[i + 2]) / 6
  elif i == 5:
     pred_elev = (his_data[i] + 2 * predict_data[i +
1] + 3 * predict_data[i + 2]) / 6
  else:
     pred_elev = (predict_data[i] + 2 *
predict_data[i + 1] + 3 * predict_data[i + 2]) / 6
  if i < 3:
     predict_data[i + 3] = pred_elev
  else:
     pred_elev = pred_elev / (1 + err)
     predict_data.append(pred_elev)
  # print(pred_elev)
  if (12 * 3 + 6) <= i <= (12 * 4 + 5):
     year_2025.append(pred_elev)
  elif (12 * 8 + 6) <= i <= (12 * 9 + 5):
     year_2030.append(pred_elev)
  elif (12 * 28 + 6) <= i <= (12 * 29 + 5):
     year_2050.append(pred_elev)
  else:
     pass
```

3. Python Code for Adaptive Filtering

```
import pandas as pd
import numpy as np
# Reading information
nclos = list(range(13))[1:]
data_2005_skip = 71
nrows_2005 = 16
elevation_data =
pd.read_excel('./2021_HiMCM_LakeMead_Monthl
```

```
yElevationData.xlsx', usecols=nclos,
skiprows=data_2005_skip, nrows=nrows_2005)
data = elevation_data.values
data_vector = data.flatten()
num_data = data_vector.shape[0]
weight = np.ones(3) * (1 / 3)
k_list = np.linspace(1e-11, 1e-9, num=20)
err_diff = 100
max_iter = 1e5
iter_time = 0
err_rate_list = []
# Finding Weights
for j in range(len(k_list)):
  k = k_list[j]
  while err_diff > 0.1 and iter_time < max_iter:
    for i in range(num_data - 3):
      y_predict = np.sum(data_vector[i: i + 3] *
weight)
      err = data_vector[i + 3] - y_predict
      err_rate = np.absolute(err) / data_vector[i +
3]
      err_rate_list.append(err_rate)
      weight += 2 * k * err * data_vector[i: i + 3]
      if i == 0:
        max_err = np.absolute(err)
        min_err = np.absolute(err)
      elif max_err < np.absolute(err):
        err_diff = np.absolute(max_err -
np.absolute(err))
        max_err = np.absolute(err)
      elif min_err > np.absolute(err):
        min_err = np.absolute(err)
    iter_time += 1
  mean_err_k = np.mean(err_rate_list)
  if j == 0:
    mean_err_all = mean_err_k
    max_err_all = max_err
    min_err_all = min_err
    k_all = k
  elif mean_err_all > mean_err_k:
    mean_err_all = mean_err_k
    max_err_all = max_err
    min_err_all = min_err
    k_all = k
  print(mean_err_k)
print("err_diff:", err_diff)
print("mean err rate:", np.mean(err_rate_list))
print("min_err:", min_err_all)
print("max_err: ", max_err_all)
print("weights: ", weight)
# Predicting
data = np.array([1067.65, 1067.96, 1067.68])
weight = np.array([0.32930616, 0.33265291,
0.33743479])
year_2025 = []
year_2030 = []
year_2050 = []
total_months = (2050 - 2021) * 12
for i in range(total_months):
  pred_elev = np.sum(data[i:i + 3] * weight)
  data = np.append(data, pred_elev)
if (12 * 3) <= i <= (12 * 4 - 1):
   year_2025.append(pred_elev)
  elif (12 * 8) <= i <= (12 * 9 - 1):
    year_2030.append(pred_elev)
  elif (12 * 28) <= i <= (12 * 29 - 1):
    year_2050.append(pred_elev)
  else:
    pass
```

4. Python Code for Non-linear Fitting

```
import numpy as np
import pandas as pd
import scipy.optimize as optimize
from datetime import datetime
import matplotlib.pyplot as plt
import matplotlib.dates as mdates
pi = np.pi
data_2021 = np.array(
  [1094.68, 1096.27, 1098.59, 1096.39, 1091.32,
1087.07, 1084.63, 1084.04, 1083.21, 1081.88,
1081.07, 1083.72])
month = np.arange(1, 13, 1)
fig, ax = plt.subplots()
def target_func(x, a0, a1, a2, a3):
  return a0 * np.sin(a1 * x + a2) + a3
# Fitting Sin
fs = np.fft.fftfreq(len(month), month[1] - month[0])
Y = abs(np.fft.fft(data_2021))
freq = abs(fs[np.argmax(Y[1:]) + 1])
a0 = max(data_2021) - min(data_2021)
[a1, a2, a3] = [2 * pi * freq, 0, np.mean(data_2021)]
p0 = [a0, a1, a2, a3]
para, _ = optimize.curve_fit(target_func, month,
data_2021, p0=p0)
print(para)
y_fit = [target_func(a, *para) for a in month]
```